A Fuzzy TOPSIS Algorithm Applied for Libyan Universities

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Abstract— **In this paper we explore some of the applications of Fuzzy TOPSIS method by using it in the problem of the academic member selection in Libyan universities. We introduce a common theme between applied mathematical and business management. We illustrate with example the ambiguity of concepts that are associated with the judgments of the human being could be solved by the Fuzzy numbers and the order of the alternatives can be determined by introducing a closeness coefficient to calculate the distances to the Fuzzy positive and negative ideal solutions using FPIS and FNIS respectively.**

***Key Words*—Fuzzy TOPSIS Method, Fuzzy Set, the Fuzzy positive and negative ideal solutions.**

I. INTRODUCTION

TO select the best candidate of the academic members to a certain faculty is a multi-criteria decision-making problem [1]. The selection committee must do a great deal of consideration to select a new academic member. The first thing they must do is advertise about them need in the social media means as an internet or a newspaper. The candidates respond and apply for the announced position. The note of the selection committee is to select the most suitable candidate for the university among those who applied [2]. The criteria that the committee is looking for are the scientific degree, personal criterion, social criteria, teaching skills, the previous experiences published papers and authored books and the research done by the candidate [3]. The collected data including the subjective criteria and the weights of the criteria of the suitable candidate are translated into linguistic terms. And so, the Fuzzy TOPSIS method has been applied in this paper while the rating of the various alternatives and the weights of all criteria are evaluated in linguistic terms that are represented by Fuzzy numbers. Hwang and toon [4], were the first to introduce the TOPSIS method. According to this method the best alternative would be the one that is the nearest to the positive ideal solution and the farthest from the negative ideal solution [5]. The positive ideal solution is one that makes the benefit criteria to be the maximum and makes the cost criteria to be the minimum, and vice versa is true with the negative ideal solution [6]. In this study we have considered the extended TOPSIS method proposed originally by chin [7]. Decision makers prefer interval judgments that fixed value judgments [8], because in classical TOPSIS the ratings and the weights of the criteria are exactly known while in real life situations this is not true since human judgments are often difficult for the decision makers to estimate their preferences with exact numerical values. So, triangular Fuzzy numbers are used by the decision makers to express the linguistic ratings. A Fuzzy decision matrix is formed by converting the linguistic variables into triangular Fuzzy numbers. The decision makers can choose the best candidate according to the values obtained by defining the FPIS and FNIS and the closeness of each alternative is calculated.

II. PRELIMINARIES

In this section we elaborate on some necessary language that is adopted from [9] and [10] We always hear uncertain expressions in our daily life like not very clear, may be so, perhaps, and very or not very likely, and so on. So, the decision makers may get misread results if the fuzziness of human decision making is not considered [11]. Zadeh [12] introduced a theory called the Fuzzy set theory to deal with the vagueness of the human thought. The fuzzy set is on extension of a crisp sets which only allow full membership or no membership at all, fuzzy sets on the other side allow partial membership. The classical set theory is a set of which is either a member or not a member. A very exact and sharp boundary exist in this theory to show that if an entity belongs to the set. Many real-life applications cannot be full filled by the classical set theory [13]. Zadeh introduced values rating from 0 to 1 to show the membership of the objects in a fuzzy set. The zero represents complete non-membership, while One represents a complete membership Values between zero and one represent intermediate degrees of membership [14]. A fuzzy set in a universe of discourse X is characterized by a membership function $\mu_{\tilde{n}}(x)$ that maps each element in X to a real number in the interval [0, 1]. A convex of fuzzy set is called a fuzzy number expressed by a grade of membership ranging from 0 to 1 [15]. In practice triangular and trapezoid fuzzy numbers are used [16]. Triangular fuzzy numbers (TFNs). These numbers are often use suitable to work with because of their computational simplicity. So, we have adopted the TFNs in the fussy TOPSIS in this study. Figure 1 shows a triangular fuzzy number.

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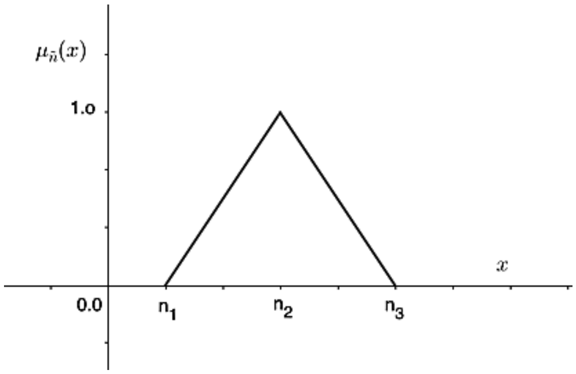

Fig. 1. Triangular Fuzzy Number.

The parameters n_1, n_2 and n_3 indicate the smallest value, the most promising value and largest value respectively. A tidle (~) will be placed above a symbol if the symbol represents a fuzzy set. The membership function is expressed as:

$$\begin{cases} 0 & if \quad x < n_1 \\ \frac{x-n_1}{n_2-n_1} & if \quad n_1 \le x \le n_2 \\ \frac{x-n_3}{n_2-n_3} & if \quad n_2 \le x \le n_3 \\ 0 & if \quad x > n_3 \end{cases} \quad (1)$$

If we define two positive TFNs $\hat{n}$ and $\hat{m}$ by the triplets $\hat{n} = (n_1, n_2, n_3)$ and $\hat{m} = (m_1, m_2, m_3)$ then,

$$\hat{n} \oplus \hat{m} = (n_1, n_2, n_3) \oplus (m_1, m_2, m_3) \quad (2)$$
$$= (n_1 + m_1, n_2 + m_2, n_3 + m_3)$$
$$\hat{n} \otimes \hat{m} = (n_1, n_2, n_3) \otimes (m_1, m_2, m_3)$$
$$= (n_1 + m_1, n_2 + m_2, n_3 + m_3) \quad (3)$$

For any positive real number k we have

$$(n_1, n_2, n_3) \otimes k = (kn_1, kn_2, kn_3)$$

The distance between two triangular fuzzy numbers can be calculated by using vertex method

$$d_v(\hat{m}_1 \hat{n}) = \sqrt{\frac{1}{3}(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2} \quad (4)$$

Remark:
If $d_v(\hat{m}_1 \hat{n}) = 0$,then the two triangular fuzzy numbers are the same numbers.

The Fuzzy TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution) can be used to evaluate multiple alternatives against the selected criteria. In this study the importance weights of various criteria and ratings of qualitative criteria are taken into accouter as linguistic variable. For decision markers the linguistic assessments are suitable for the subjective judgment. The triangular fuzzy numbers have been used to capture the vagueness of the linguistic assessments. Figures 2 and 3 show the linguistic variable used by the decision markers to evaluate the importance of the criteria and the ratings of the alternatives with respect to criteria [17].

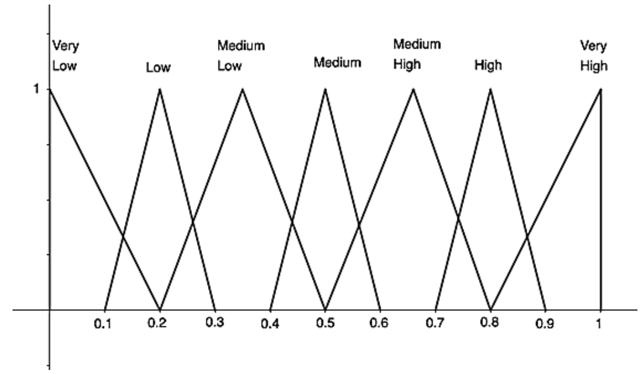

Fig. 2. Linguistic Variables for Importance Weight of each Criterion.

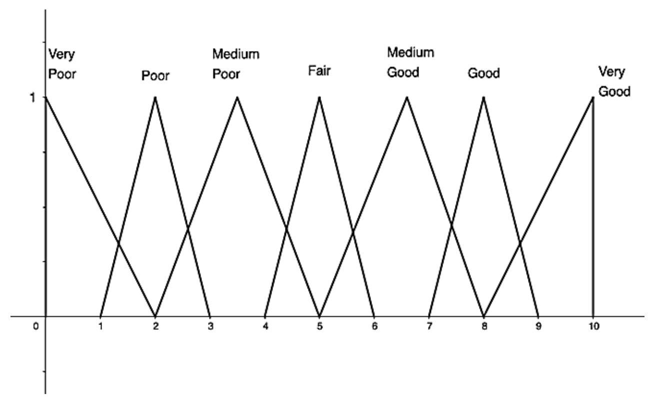

Fig. 3. Linguistic Variables for Rating.

In this part we will learn about the Fuzzy TOPSIS method. As well as becoming familiar with the notation and symbols used in the Fuzzy TOPSIS see [17].

1) We denote by $E = \{D_1, D_2, \dots \dots D_K\}$ the set of K decision makers.
2) We denote by A=$\{A_1, A_2, \dots \dots A_m\}$ the set of possible alternatives.
3) We denote by C=$\{C_1, C_2, \dots \dots C_n\}$ the set of n criteria with which alternative performances n are measured.
4) We denote by $X = \{x_{ij}\}$the set of performance ratings of A_i with respect to criteria C_j where $i = 1,2, \dots., m \; and \; j = 1,2, \dots., n$

For any decision makers fuzzy rating D^t where t=1,2, ….., k can be represented as triangular fuzzy number $R^t = (a^t, b^t, c^t)$ with membership function $\mu_{\tilde{n}}(x), moreover$ the aggregated fuzzy rating can be defined as $\tilde{R} = (a, b, c)$ where

$$a = min\{a_{ij}^t\} \quad b = \frac{1}{k}\sum_{t=1}^{k} b_{ij}^t \quad c = \max\{c^t\} \quad (5)$$

For any fuzzy rating of decision $\tilde{x}_{ij}^{t}$=($a_{ij}^t, b_{ij}^t, c_{ij}^t$) the aggregated fuzzy rating of decision of alternative with respect to each criterion can be defined as

$\tilde{x}_{ij}^{t} = (x_{ij}, x_{ij}, x_{ij})$ where

$$a_{ij} = min\{a_{ij}^t\} \quad b_{ij} = \frac{1}{k}\sum_{t=1}^{k} b_{ij}^t \quad c_{ij} = max\{c_{ij}^t\} \tag{6}$$

The aggregated fuzzy importance weight of each criterion can be defined as

$\widetilde{w_j} = (w_{j1}, w_{j2}, w_{j3})$ where

$$w_{j1} = min\{w_{j1}^t\} \; w_{j2} = \frac{1}{k}\sum_{t=1}^{k} w_{j2}^t \; w_{j3} = max\{w_{j3}^t\} \tag{7}$$

We can abbreviate academic member selection problem with this matrices [15]

$$\widetilde{D} = \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & & \\ \vdots & & \ddots & \vdots \\ \tilde{x}_{m1} & & \cdots & \tilde{x}_{mn} \end{pmatrix} \tag{8}$$

And

$$\widetilde{w_j} = (\widetilde{w}_1, \widetilde{w}_2, \widetilde{w}_3) \tag{9}$$

Let B and F be the set of benefit criteria and cost criteria respectively , then the normalized fuzzy decision matrix defined as $\tilde{P} = [\tilde{p}_{ij}]_{n \times m}$ where

$$\tilde{p}_{ij} = \begin{cases} \left(\dfrac{a_{ij}}{max\{c_{ij}\}}, \dfrac{b_{ij}}{max\{c_{ij}\}}, \dfrac{c_{ij}}{max\{c_{ij}\}}\right) & if \;\; j \in B \\ \left(\dfrac{min\{a_{ij}\}}{c_{ij}}, \dfrac{min\{a_{ij}\}}{b_{ij}}, \dfrac{min\{a_{ij}\}}{a_{ij}}\right) & if \;\; j \in F \end{cases} \tag{10}$$

The weighted normalized fuzzy decision matrix is formed as $\tilde{V} = [\tilde{v}_{ij}]_{n \times m}$

The fuzzy positive ideal solution (FPIS) A^* and the fuzzy negative ideal solution (FNIS) A^- can be defined as

$$A^* = \{\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*\} \tag{11}$$

$$A^- = \{\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-\} \tag{12}$$

Where $\tilde{v}_j^* = max\{v_{ij3}\}$ and $\tilde{v}_j^- = min\{v_{ij1}\}$. The formula for calculating the sum of the distance between each of alternative from A^* and $\tilde{v}_j^*$ is

$$d_i^* = \sum_{j=1}^{n} d_v(\tilde{v}_{ij}, \tilde{v}_j^*) \tag{13}$$

The formula for calculating the sum of the distance between each of alternative from A^* and $\tilde{v}_j^-$ is

$$d_i^- = \sum_{j=1}^{n} d_v(\tilde{v}_{ij}, \tilde{v}_j^-) \tag{14}$$

The closeness coefficient CC_I represents the distances to fuzzy PIS A^* and the fuzzy NIS A^- simultaneously . The closeness coefficient of each alternative is calculated as

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \tag{15}$$

III. Applications of Fuzzy TOPSIS

According to what how been mentioned in previous sections, working team from the business management departments in Gulf of Sidra University and mathematics departments in University of Derna and University of Bright Star, was formed to choose the appropriate academic staff member with a high efficiency, according to criteria suggested by the business management and department, and to follow the Fuzzy TOPSIS Method suggested by the department mathematics. The criteria of choosing an academic staff member

1) Personal criterion C_1. This is represented in the personality power, general appearance, having linguistic energy and ability to express clearly, and to be healthy enough to be able to achieve his job, with quick thought and reply, in addition to good behavior at difficult positions. He should have a big self-trust straight, honest, and faithful
2) Social criterion C_2. This is represented in good dealing with the students, build relation of friendship with them, and encourage them to learn.
3) Teaching skills C_3.
4) Teaching experience C_4.
5) Publications and Research C_5.

The following tables plays an important role in showing the Fuzzy TOPSIS Method. The importance weights of the criteria determined by three decision makers are shown in Table 1. The rating of five candidates under five criteria are shown in Table 2.

Table 1: Importance Weight of Criteria from three Decision Makers

	Decision Makers		
Criteria	D_1	D_2	D_3
C_1	M	MH	H
C_2	H	MH	MH
C_3	MH	H	VH
C_4	VH	VH	H
C_5	H	H	MH

Table 2: Rating of Five Candidates by Decision Makers

		Decision Makers		
Criteria	Alternative	D_1	D_2	D_3
C_1	A_1	MP	MP	F
	A_2	P	MP	F
	A_3	F	MG	G
	A_4	MG	G	VG
	A_5	MP	MG	F
C_2	A_1	F	MP	O
	A_2	MG	MP	F
	A_3	G	F	MG
	A_4	VF	MP	MP
	A_5	VG	MG	G
C_3	A_1	MG	MP	F
	A_2	F	MP	MP
	A_3	F	P	MP
	A_4	G	F	MG
	A_5	G	G	MG
C_4	A_1	VG	G	MG
	A_2	G	MG	F
	A_3	F	MP	MP
	A_4	F	MP	P
	A_5	G	MG	F
C_5	A_1	MG	G	F
	A_2	MP	F	MP
	A_3	MG	G	F
	A_4	F	F	MG
	A_5	G	VG	MG

We convert Table 1 and Table 2 to form fuzzy decision matrix as shown in Table 3.

Table 3: Fuzzy Decision Matrix and Fuzzy Weighted of Five Candidates

	A_1	A_2	A_3	A_4	A_5	Weight
C_1	(2,4,6)	(1,3,5,6)	(4,6,5,9)	(5,8,2,10)	(2,5,8)	(0.4,0.65,0.9)
C_2	(1,3,5,6)	(2,5,8)	(4,6,5,9)	(2,4,6)	(5,8,2,10)	(0.5,0.7,0.9)
C_3	(2,4,8)	(2,4,6)	(1,3,5,6)	(4,6,5,9)	(5,7,5,9)	(0.5,0.82,1)
C_4	(5,8,2,10)	(4,6,5,9)	(2,4,6)	(1,3,5,6)	(4,6,5,9)	(0.7,0.9,1)
C_5	(4,6,5,9)	(2,4,6)	(4,6,5,9)	(4,5,5,8)	(5,8,2,10)	(0.5,0.7,0.9)

The normalized Fuzzy decision matrix is formed as in Table 4.

Table 4: Normalized Fuzzy Decision Matrix

	A_1	A_2	A_3	A_4	A_5
C_1	(0.2,0.4,0.6)	(0.1,0.35,0.6)	(0.4,0.65,0.9)	(0.5,0.82,1)	(0.2,0.5,0.8)
C_2	(0.1,0.35,0.6)	(0.2,0.5,0.8)	(0.4,0.65,0.9)	(0.2,0.4,0.6)	(0.5,0.82,1)
C_3	(0.2,0.4,0.8)	(0.2,0.4,0.6)	(0.1,0.35,0.6)	(0.4,0.65,0.9)	(0.5,0.75,0.9
C_4	(0.5,0.82,1)	(0.4,0.65,0.9)	(0.2,0.4,0.6)	(0.1,0.5,0.6)	(0.4,0.65,0.9
C_5	(0.4,0.65,0.9)	(0.2,0.4,0.6)	(0.4,0.65,0.9)	(0.4,0.55,0.8)	(0.5,0.82,1)

Then weighted normalized Fuzzy decision matrix is formed as in Table 5.

Table 5: Weighted Normalized Fuzzy Decision Matrix

	A_1	A_2	A_3	A_4	A_5
C_1	(0.08,0.26,0.54)	(0.04,0.23,0.54)	(0.16,0.43,0.81)	(0.2,0.54,0.9)	(0.08,0.33,0.72)
C_2	(0.05,0.23,0.54)	(0.1,0.35,0.72)	(0.24,0.46,0.81)	(0.1,0.28,0.54)	(0.25,0.58,0.9)
C_3	(0.1,0.33,0.8)	(0.1,0.33,0.6)	(0.05,0.29,0.6)	(0.42,0.54,0.9)	(0.25,0.62,0.9)
C_4	(0.35,0.74,1)	(0.28,0.59,0.9)	(0.14,0.36,0.6)	(0.07,0.45,0.6)	(0.28,0.59,0.9)
C_5	(0.2,0.46,0.81)	(0.1,0.28,0.54)	(0.2,0.46,0.81)	(0.2,0.39,0.72)	(0.25,0.58,0.9)

After forming weighted normalized Fuzzy decision matrix, we determine FPIS and FNIS.

A^* ={(0.9,0.9,0.9), (0.9,0.9,0.9), (0.9,0.9,0.9), (1,1,1), (0.9,0.9,0.9)}
A^-={(0.04,0.04,0.04),(0.05,0.05,0.05),(0.05,0.05,0.05),(0.07,0.07,0.07), (0.1,0.1,0.1)}

Therefore, the distance between each of alternative from FPIS and FNIS are calculated as:

$$d\,(A_1,A^*) = \sqrt{\tfrac{1}{3}[(1-0.08)^2 + (0.04-0.26)^2 + (0.04-0.54)^2]} = 0.73$$

$$d\,(A_1,A^-) = \sqrt{\tfrac{1}{3}[(0.04-0.08)^2 + (0.04-0.26)^2 + (0.04-0.54)^2]} = 0.32$$

The results of all alternative distance from FPIS and FNIS are shown in Table 6 and Table 7

Table 6: distances between A_i $(i = 1,2,3,4,5)$and A^* with respect to each criterion

	$d(A_1,A^*)$	$d(A_2,A^*)$	$d(A_3,A^*)$	$d(A_4,A^*)$	$d(A_5,A^*)$
C_1	0.73	0.58	0.60	0.54	0.68
C_2	0.76	0.67	0.55	0.72	0.50
C_3	0.66	0.69	0.73	0.44	0.49
C_4	0.41	0.49	0.66	0.67	0.48
C_5	0.51	0.72	0.57	0.61	0.50

Table 7: distances between A_i $(i = 1,2,3,4,5)$and A^- with respect to each criterion

	$d(A_1,A^-)$	$d(A_2,A^-)$	$d(A_3,A^-)$	$d(A_4,A^-)$	$d(A_5,A^-)$
C_1	0.32	0.31	0.51	0.58	0.42
C_2	0.31	0.44	0.52	0.33	0.60
C_3	0.47	0.37	0.36	0.62	0.61
C_4	0.71	0.61	0.38	0.41	0.61
C_5	0.52	0.33	0.52	0.52	0.60

Thus, the closeness coefficient of five alterative are calculated as

$$CC_1 = \frac{2.33}{3.13+2.33} = 0.43$$

$$CC_2 = \frac{2.06}{3.15+2.06} = 0.40$$

$$CC_3 = \frac{2.29}{3.11+2.29} = 0.44$$

$$CC_4 = \frac{2.46}{5.44} = 0.46$$

$$CC_5 = \frac{2.84}{2.65+2.84} = 0.52$$

According to the closeness coefficient of five alterative, the order of five alterative is $A_5 > A_4 > A_3 > A_1 > A_2$. Therefore we will choose the alterative A_5

IV. Conclusion

The use of fuzzy concepts in decision making is considered appropriate. To get the ideal solution, the appropriate negation and design the TOPSIS model for the fuzzy environment were introduced. The lower bound of the choices to apply a new measurement of the fuzzy distance value is used for best result selected. Then use the similarity to rank your choices. an example is shown to show the functionality of the proposed model. Fuzzy decision matrix is formed by converting the linguistic variables into triangular fuzzy numbers. The decision makers can choose the best candidate according to the values obtained by defining FPIS and FNIS and the closeness of each alternative is calculated. the solved problem of the academic member selection is presented based the technique of applied fuzzy mathematical modeling. The application applied for some academic staff at varies universities.

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